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Underdetermined blind identification using a generalized generating function, tucker decomposition, and alternating least squares

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Abstract

Blind identification of signals with actual values has made use of the generating function (GF). In order to better leverage the statistical information conveyed on complex signals, this study first generalizes the definition of GF to include complex-valued random variables. Next, we propose an algebraic framework for determining the mixing matrix from underdetermined mixtures by means of the generalized generating function (GGF). For this, we create two new techniques: GGF-ALS and GGF-TALS. The GGF-ALS technique uses an alternating least squares (ALS) algorithm to estimate the mixing matrix through the decomposition of the tensor built from the Hessian matrices of the GGF of the data. As a refined iteration of the original Tucker decomposition-based GGF-ALS algorithm, the GGF-TALS approach has many advantages. To be more precise, the Tucker decomposition is used to reduce the rank of the original tensor created in GGF-ALS, using the components acquired from the left singular-value decomposition of the mode-3 matrix of the original tensor. The ALS approach is then used to estimate the mixing matrix by decomposing the core tensor. The proposed GGF-ALS and GGF-TALS methods perform nearly as well in terms of the relative errors as the state-of-the-art GF-based baseline methods, but the GGF-TALS has much lower computational complexity, as shown by the simulation results, and (b) the proposed GGF algorithms perform better than the state-of-the-art GF-based baseline methods.

Key words :

Underdetermined mixtures; tensor decomposition; tucker decomposition; blind identification; generalized generating function.

Introduction

Recent years have seen a surge in study into blind identification (BI) of linear mixes, which has applications in many areas of signal processing, including blind source separation (.BSS). This study focuses on BI for uncertain combinations whose origins are difficult to pin down. The receiving of more sources than sensors become more achievable with the expansion in reception bandwidth, making underdetermined mix trues a typical occurrence in many real-world applications like radio communication contexts. Complex sources are another common challenge in these contexts. Quadrature amplitude modulation (QAM) and minimum-shift keying (MSK) transmissions are examples of complex-valued communication signals. Also, objective functions used in the frequency domain are typically defined on complex-valued variables, which makes them well-suited for blind separation and identification from convolutive mix trues [1,2]. Many BI approaches for underdetermined mixtures assume that the sources are either naturally sparse (in their own domain, like the time domain) or can be made sparse (in a transform domain, for example). If the signal is not sparse by definition, it is common practice to apply a predetermined transform, such the short-time Fourier transform (STFT), or a learnt transform utilizing, for instance, simultaneous codeword optimization (Simcoe), in order to sparsity the data [3,4].

High signal levels in the directions of the mixing vectors, which can be localized using the scatter plot, are usual since the sources are relatively few in number. bear in mind that although certain signals, like voice signals, do exhibit a degree of sparsity in one domain or another, many other signals, like the vast majority of communication signals, do not. As a result, it is important to create BI techniques for the underdetermined mixes that do not need the sources to be sparse. Decomposition techniques based on distinct data structures, such correlation [7,8] and higher-order cumulant [9–14] matrices, are often used in approaches for BI of underdetermined mixtures to achieve this goal. The fundamental concept behind these techniques is to first estimate the mixing matrix by decomposing a tensor built from the cumulants of the data. This is especially true for algorithms like the Second-Order Blind Identification of Underdetermined Mixtures (SOBIUM) [7], the

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Fourth-Order Blind Identification of Underdetermined Mixtures (FOBIUM) [9], the Fourth-Order-Only Blind Identification of Underdetermined Mixtures (FOOBI) [10], the Fourth-Order-Only Blind Identification of Underdetermined Mixtures (FOOBI-2) [10], and the Blind Identification of Mixtures of Sources Using Redundancies in the data Based on the even-order cumulants of the observations, a family of approaches is proposed in [13] with the term blind identification of over-complete mixtures of sources (BIOME). The data measured by second-order or higher-order statistics are the only ones used by any of the techniques provided in [7-14].

Definition of the Issues Taking into account the linear mixing model shown below

$$\mathbf{z}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t)$$

where the stochastic vector z(t) ∈ C Q represents the observation signals, s(t) ∈ C P contains the unobserved source signals, and w(t) ∈ C Q denotes additive noise. From now on, the noise w(t) is simply ignored for convenience, except when running computer experiments. The unknown mixing matrix A ∈ C Q × P characterizes the way that the sources are acquired by the sensors. BI aims to estimate the mixing matrix from the observations based on the assumption that the source signals are statistically independent. The mixing matrix obtained may in turn be used to estimate the original source signals from the observations. In addition, we make the following assumptions: (i). The mixing matrix A is of full (row) rank. (ii).The number P of sources is known. (iii). The number of sensors is smaller than the number of sources, i.e., Q<P.

Algebraic structure based on generalized generating function 3.1. Core equation based on generalized generating function For a real stochastic vector x ∈ R Q, the GF ϕx(u) obtained by dropping the term of the square root of (−1) in the exponent of a CAF is defined as

$$\phi_x(\mathbf{u}) = \mathrm{E}\left[\exp\left(\mathbf{u}^{\mathrm{T}}\mathbf{x}\right)\right], \mathbf{u} \in \mathrm{R}^{\mathrm{Q}},$$

where u ∈ R Q is an arbitrary vector referred to as a processing point [15], and E[ ] denotes an expectation operator. Nevertheless, both the observation vector z and the mixing matrix A discussed in this paper belong to the complex field. Hence, a definition of GF for complex variables is required. One such definition has been presented in [18] as

$$\begin{aligned}\phi_z(\Re(\mathbf{u}), \Im(\mathbf{u})) &= \mathrm{E}\left[\exp\left(\Re\left(\mathbf{u}^{\mathrm{H}}\mathbf{z}\right)\right)\right] \\ &= \mathrm{E}\left[\exp\left(\Re^{\mathrm{T}}(\mathbf{u})\Re(\mathbf{z}) + \Im^{\mathrm{T}}(\mathbf{u})\Im(\mathbf{z})\right)\right], \mathbf{u} \in C^{\mathrm{Q}},\end{aligned}$$

where Rð Þ· and Ið Þ· denote taking the real and imaginary parts from their arguments (i.e., complex-valued vectors) to form a real-valued vector of the same dimension. It is actually defined by assimilating C to R2 . Thus the GF of a complex variable in (3) is defined as a function of the real and imaginary parts. In this paper, we generalize the definition of GF for real stochastic vector in (2) to the following complex from

$$\psi_z(\mathbf{u}) = \mathrm{E}\left[\exp\left(\mathbf{u}^{\mathrm{H}}\mathbf{z}\right)\right], \mathbf{u} \in C^{\mathrm{Q}}$$

Note that the statistical information exploited by GF/ GGF is related to the number of processing points. Theoretically, a complete statistical description of the probability density function requires the evaluation of the GF/GGF at all (infinitely many) possible processing points. However, this often becomes computationally infeasible. In practice, such statistical information is obtained approximately by the evaluation of GF/GGF at a finite number of processing points. Hence, in comparison with the GF presented in [18], the GGF defined in (4) can exploit the statistical information carried on the complex variables more effectively when the number of the processing points stays the same, thanks to the incorporation of the imaginary part of the exponent to the function. Furthermore, as compared with the use of the GF in (3), using the GGF in (4) offers a simpler way for the estimation of the mixing matrix due to the exploitation of an elegant algebraic structure. Now, replacing z by its model and neglecting the noise contribution yield

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$$\psi_z(\mathbf{u}) = \mathrm{E}\left[\exp\left(\mathbf{u}^{\mathrm{H}}\mathbf{A}\mathbf{s}\right)\right] = \psi_s\left(\mathbf{A}^{\mathrm{H}}\mathbf{u}\right), \mathbf{u} \in \mathrm{C}^{Q}.$$

Defining

$$\phi_z(\mathbf{u}) = \log \psi_z(\mathbf{u}),$$

which is often referred to as the 'second' GGF, and using the source independence

property, the second GGF of the observations can be rewritten as

$$\varphi_z(\mathbf{u}) = \sum_{p=1}^{P} \varphi_{s_p}\left(\mathbf{a}_p^{\mathrm{H}}\mathbf{u}\right)$$

Consequently, by calculating the derivative of the conjugate gradient of φz(u) with respect to u (more details can be found in Appendix 1), we can obtain the following core equation for the Hessian matrix ψz(u)

$$\boldsymbol{\psi}_z(\mathbf{u}) = \mathbf{A}\boldsymbol{\psi}_s\left(\mathbf{A}^{\mathrm{H}}\mathbf{u}\right)\mathbf{A}^{\mathrm{H}}$$

Wich

$$\boldsymbol{\psi}_z(\mathbf{u}) = \nabla_{\mathbf{u}^{\mathrm{T}}}\left[\nabla_{\mathbf{u}^*}\varphi_z(\mathbf{u})\right] = \frac{\partial}{\partial \mathbf{u}^{\mathrm{T}}}\left[\frac{\partial \varphi_z(\mathbf{u})}{\partial \mathbf{u}^*}\right],$$

The conjugate operator is shown by ()*. It's important to note that s(AHu) is a diagonal matrix (see Appendix 2 for further information).

Tensor decomposition-based blind identification

The GGF-ALS and GGF-TALS algorithms are constructed in this part to estimate the mixing matrix. The Hessian matrices of the GGF of the data are decomposed to estimate the mixing matrix in the GGF-ALS technique. The GGF-TALS algorithm, which is an enhancement of the GGF-ALS method, is also created.

the GGF-ALS algorithm's original tensor is converted to a lower-rank core tensor using Tucker decomposition, and then the mixing matrix is computed via ALS decomposition of the core tensor.

Analysis and simulations

Here, we give simulations to demonstrate how the suggested GGF methods fare when applied to uncertain mixes of complicated sources. Algorithms are measured and compared based on their relative error performance index (PI) in relation to the sample size and the signal-to-noise ratio (SNR) of the observations. Here we define the relative error PI as [7] PI 14 EfjjAA jj=jjAjjg, where A is the ideally or dered and scaled approximation of the mixing matrix, and fjj is the Frobenius norm. The tests assume a uniform circular array (UCA) with Q = 3 identical sensors of radius Ra receiving signals from a P = 4 nar rowband source. The elements of the mixing matrix A for a free-space propagation model are

$$a_{qp} = \exp\left(2\pi j\left(\alpha_q \cos(\theta_p)\cos\left(\phi_p\right) + \beta_q \cos(\theta_p)\sin\left(\phi_p\right)\right)\right)$$

given that q = (Ra/)cos(2(q 1)/Q), q = (Ra/)sin(2(q 1)/Q, and j 14 ffi1 p. Ra/ = 0.55 is the result. The DOAs of the various sources are provided by (1, 3, 10, 2, 5, 0), (1, 7, 9, 10, 3, 3, 4, 4) and (1, 7, 10, 2, 3, 4, 4) respectively. The filters have a roll-off of = 0.3 and are designed to shape 4-QAM sources with a unit-variance distribution. The length of each source's symbols is constant: T = 4Te, where the is the sampling interval. Zero-mean complex additive Gaussian noise is present in the observations. To start, we analyse the impact of the rank of the core tensor L on the performance of the GGF-TALS algorithm by comparing it to that of the GGF-ALS method. This is investigated using two separate simulation studies. In the first simulation, we compare the performance of GGF-ALS and GGF-TALS when both algorithms are run with the same number of processing points K but with varying ranks for their core tensors L. The simulation runs with 4,000 samples and a signal-to-noise ratio (SNR) of 0-25 dB, with K set to 100 for the GGF-ALS algorithm. The real and imaginary portions of

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processing points are picked at random from [1; 1]. For the GGF-TALS method, the core tensor rank is set at L = 10,8,5. The ALS method is terminated at the threshold value of 105 as indicated in (12), and 100 Monte Carlo simulations are performed. Figure 2 displays the average performance of the GGF-TALS algorithm in relation to the core tensor rank for a constant number of processing points. Evidently, the

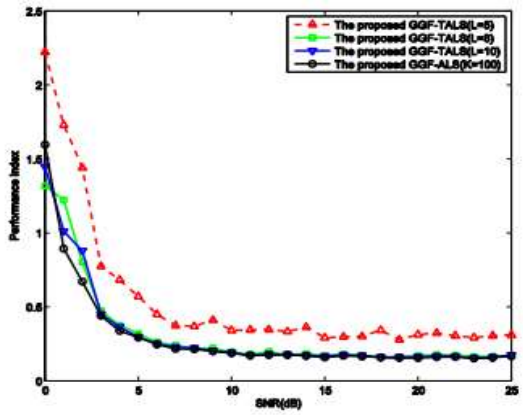

Figure 2 Performance of GGF-TALS versus core tensor order for a fixed number of processing points. The results for GGF-ALS are also shown for comparison.

There is no difference between the performance curves of GGF-TALS and GGF-ALS for core tensor ranks more than 8, however the GGF-TALS method suffers from a little drop in performance for core tensor ranks fewer than 8. The identification accuracy provided by GGF-TALS is identical to that provided by GGF-ALS, despite the fact that the core tensors in GGF-TALS have a substantially lower rank than the original tensor utilized by GGF-ALS. In the second experiment, we look at how the GGF-TALS method fares when we maintain the same rank for the core tensor L but change the number of processing points K. Except for a lower rank of the core tensor in the GGF-TALS algorithm and higher numbers of processing points in the GGF-ALS algorithm (20, 40, and 100, respectively), all other simulation parameters remain the same as before. Figure 3 shows that, for a given core tensor rank, the GGF-TALS's average performance degrades as the number of processing points grows. We can observe that the average performance of GGF-TALS is comparable to that of GGF-ALS when both algorithms employ the same number of processing points. For instance, for K = 40, GGF-TALS performs similarly to GGF ALS at 100 processing points. Thus, the GGF-TALS algorithm yields similar results to GGF-ALS regardless of the number of processing points used. Secondly, we compare the most recent GF-based BI algorithm to its two forerunners, GGF-ALS and GGF-TALS. As a point of departure, we use the LEMACFAC-2 method outlined in [18]. The overall number of processing points and per-point value are same throughout the GGF-ALS, GGF TALS, and LEMACFAC-2. There are 100 processing nodes, the core tensor has a rank of 8, and the real and imaginary parts of the nodes are picked at random from [1; 1]. The LM algorithm [26] is used in the LEMACFAC-2 method, and the associated stop value is 105 as given in (12). Figure 4 shows the PI as a function of the SNR for each of the investigated techniques, using a dataset of size 4,000 samples. As can be seen in the image, the GGF-ALS and GGF-TALS both perform better than LEMACFAC-2. You can see the proof in Figure 2.

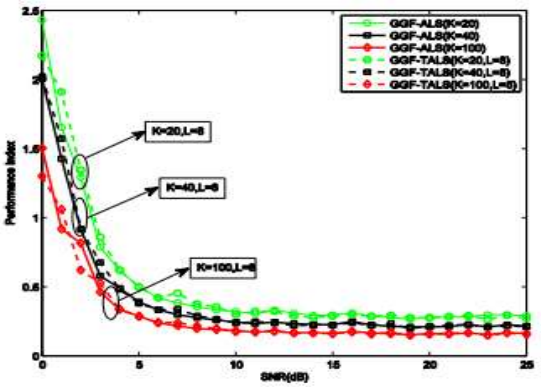

Figure 3 The performance of GGF-TALS versus the number of processing points. The results for GGF-ALS are also shown for comparison

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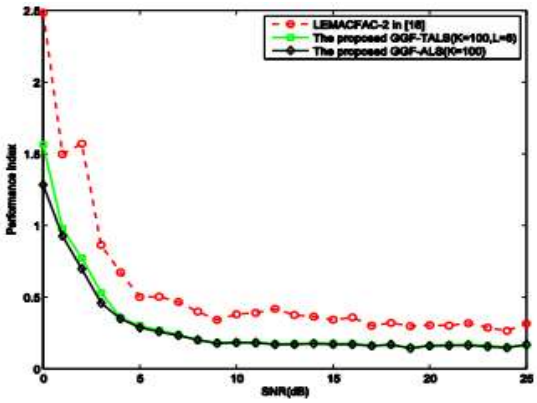

Figure 4 The performance of the tested algorithms versus the SNR of the observations.

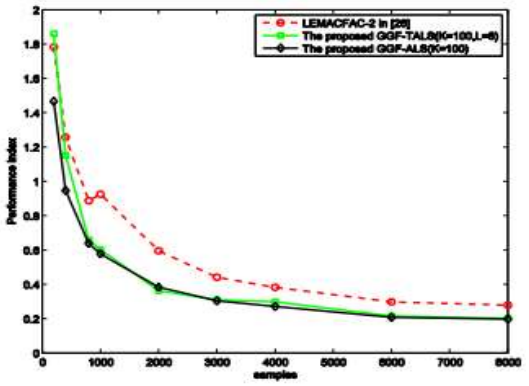

Figure 5 The performance of the tested algorithm versus the size of the data set.

the SNR (in dB) at which each method was evaluated as a function of the sample size (N). Once again, the GGF-ALS and GGF-TALS algorithms outperform LEMACFAC-2, and they almost match the performance of LEMACFAC-2. This demonstrates that, for a given number of processing points, the GGF algorithms outperform the approach based on the GF specified in (3) when it comes to making use of the statistical information contained in the complex variables.

Conclusions

We introduce two methods, GGF-ALS and GGF-TALS, that use the second GGF of the observations to perform blind identification from uncertain mixes of complicated sources. The Hessian tensor built from the second GGF of the data is used to directly deconstruct the ALS method to estimate the mixing matrix in the GGF-ALS algorithm. The GGF-TALS method is an enhanced variant of the GGF-ALS algorithm that uses the Tucker decomposition to transform the input tensor into a lower-rank core tensor before estimating the mixing matrix using the ALS technique. Based on the simulation results, (a) the proposed GGF-ALS and GGF-TALS approaches perform almost similarly in terms of the relative errors, while the GGF-TALS has a much lower computational complexity, and (b) the proposed GGF algorithms perform better than the most recent GF-based BI approaches because they better exploit the statistical information carried on complex variables.

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